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COMPUTER PROGRAM FOR STABILITY ANALYSIS OF STEEP, COHESIVE RIVERBANKS

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ABSTRACT

The ability to predict the stability of eroding riverbanks is a prerequisite for modelling alluvial channel width adjustment and a requirement for predicting bank erosion rates and sediment yield associated with bank erosion. Mass-wasting of bank materials under gravity occurs through a variety of specific mechanisms, with a separate analysis required for each type of failure. This paper presents a computer program for the analysis of the stability of steep, cohesive riverbanks with respect to planar-type failures. Planar-type failures are common along stream channels destabilized by severe bed degradation. Existing stability analyses for planar-type failures have a number of limitations that affect their physical basis and predictive ability. The computer program presented here is based on an analysis developed by Darby and Thorne. The software takes account of the geotechnical characteristics of the bank materials, the shape of the bank profile, and the relative elevations of the groundwater and surface water to estimate stability with respect to mass failure along a planar-type failure surface. Results can be displayed either in terms of a factor of safety (ratio of resisting to driving forces), or probability of failure. The computer analysis is able to determine the relative amounts of bed degradation and bank-toe erosion required to destabilize an initially stable bank. Data for the analysis are supplied in the form of either HEC-2 hydrographic survey data files or user-supplied bank profile data, in conjunction with user-supplied geotechnical parameter values. Some examples, using data from the Upper Missouri River in Montana, are used to demonstrate potential applications of the software. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: river bank erosion; bank stability; channel widening; channel stability; planar failure

INTRODUCTION

Riverbank erosion and associated sedimentation and land loss hazards are a resource management problem of global significance. In the United States, an estimated 142 000 miles (227 000 km) of streambank are in need of erosion protection, with the cost of protecting US streambanks in 1981 about US$1 billion (US Army Corps of Engineers, 1983). Recently, attempts to predict land loss and bank sediment yield associated with riverbank instability and width adjustments (Osman and Thorne, 1988; Lohnes, 1991; Darby and Thorne, 1992; Thorne and Abt, 1993) have been made. These approaches have been based on estimating the failure geometry of cohesive banks that become unstable following bed degradation and/or direct fluvial shear erosion. Failure along an approximately planar failure surface takes place when erosion of the bank and the channel bed adjacent to the bank increase the height and steepness of the bank to the point that it reaches a condition of limiting stability. Failure may also be triggered by changes in the geotechnical characteristics of the bank materials. Examples include loss of cohesion by frost weathering (Lawler, 1986), or occurrence of positive pore pressures following rapid drawdown. The development of tension cracking is also important in

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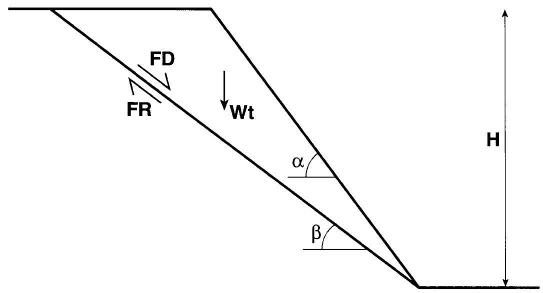
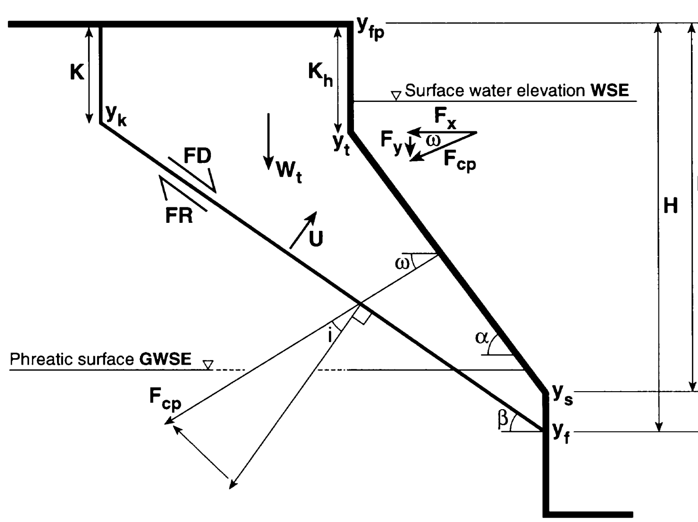


Figure 1. Idealized bank profile used in simplified bank stability analyses. Symbols: β = failure plane angle, α = uneroded bank angle, W_t = weight of failure block, FD = driving force, FR = resisting force, H = total bank height

promoting bank failures of this type (Darby and Thorne, 1994, 1997). The mechanics of failure depend on the engineering properties of the soil and the geometry of the bank at the point of collapse (Thorne and Abt, 1993).

In the past, analyses of the stability of steep, cohesive, eroding banks which fail along planar surfaces have been based on estimating the driving and resisting forces acting on incipient failure blocks using a relatively simple, idealized geometry (Figure 1). Examples of analyses based on geometries of this type include Lohnes and Handy (1968), Thorne *et al.* (1981), Huang (1983) and Simon *et al.* (1991). However, these analyses are somewhat limited when they are applied under conditions encountered in the field (Darby and Thorne, 1996a; Millar and Quick, 1997). Specifically:



$$FS = \frac{FR}{FD} = \frac{cL + (\mathbf{W_t} \cos\beta - \mathbf{U} + \mathbf{F_{cp}} \cos i)\tan\phi}{\mathbf{W_t} \sin\beta - \mathbf{F_{cp}} \sin i}$$

Figure 2. Definition diagram for the Darby–Thorne bank stability analysis (modified after Darby and Thorne, 1996a). Symbols: K = tension crack depth, K_h = relic tension crack depth, i = angle between resultant of hydrostatic confining force and normal to failure plane, β = failure plane angle, U = hydrostatic uplift force, F_{cp} = hydrostatic confining force, ω = angle at which the resultant of the hydrostatic confining force is directed, α = uneroded bank angle, $GWSE$ = groundwater surface elevation, WSE = surface water elevation, W_t = failure block weight, FD = driving force, FR = resisting force, y_{fb} = floodplain elevation, y_t = base of 'vertical face', y_s = elevation of base of uneroded bank slope (base of 'upper bank'), y_f = elevation of base of failure plane, y_k = elevation of base of tension crack, H = total bank height, H' = uneroded bank height, L = length of failure plane. The terms 'vertical face' and 'upper bank' are defined and used by Simon and Hupp (1992)

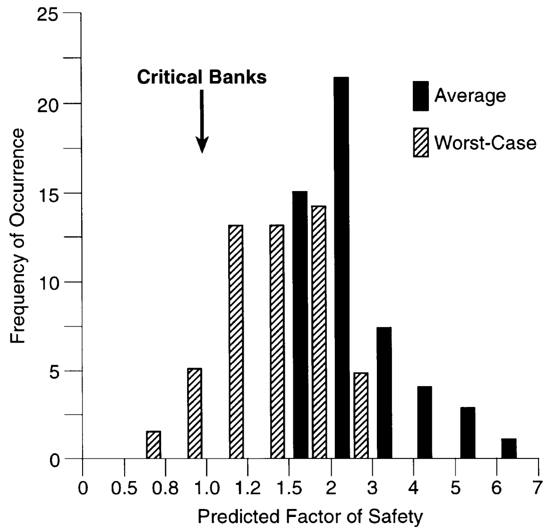


Figure 3. Assessment of the predictive ability of the bank stability analysis developed by Darby and Thorne (1996a). Predictions were obtained for 51 streambank failures in north Mississippi with factor of safety 1·0 or less

Table I. Summary of predictive ability of tested bank stability analyses (after Darby and Thorne, 1996a). Each analysis was used to calculate the factor of safety using data for 51 separate unstable streambanks. Predicted factors of safety should therefore be less than or equal to 1·0 to successfully replicate observed bank instability

Analysis	Mean predicted factor of safety ('ambient' soils)	Mean predicted factor of safety ('worst-case' soils)
Lohnes and Handy (1968)	2·69	1·83
Huang (1983)	3·68	3·26
Osman and Thorne (1988)	3·20	1·82
Darby and Thorne (1996a)	2·80	1·43

1. The idealized geometry depicted in Figure 1 is inadequate to characterize the profile of natural, eroding riverbanks, particularly when a tension crack is present (Osman and Thorne, 1988). Combinations of near-bank bed degradation and bank-toe erosion tend to result in a characteristic bank profile more akin to that shown in Figure 2.
2. The failure plane is constrained to pass through the toe of the bank. Field observations indicate this is sometimes unrealistic (Simon *et al.*, 1991).
3. The effects of soil pore water pressures and the hydrostatic confining pressure of water in the channel are usually either ignored, or characterized by a simplified pore pressure ratio term (Simon *et al.*, 1991).
4. Application of the planar failure analysis is restricted to very steep banks (Taylor, 1948; Millar and Quick, 1997).

Some of these limitations have individually, or in combination, been addressed in more sophisticated analyses. Osman and Thorne (1988) based their stability analysis on the bank profile shown in Figure 2, but they still assumed that the failure plane passes through the toe of the bank, and pore water and hydrostatic confining pressure effects were neglected. On the other hand, pore water and hydrostatic confining pressure

effects were included in an analysis developed by Simon *et al.* (1991), but in which the idealized bank profile of Figure 1 was retained. More recently Rinaldi and Casagli (1999) and Simon *et al.* (1999) have developed analyses focusing attention on the effects of negative pore water pressures in the unsaturated portion of the bank, the stabilizing effects of the hydrostatic confining pressure, and the destabilizing effects of positive pore water pressures in the saturated part of the bank. These analyses provide realistic representations of the effects of pore water pressures in the saturated and unsaturated parts of the bank profile, which is an important advance, but they still retain the simplified bank profile shown in Figure 1.

Darby and Thorne (1996a) developed a riverbank stability analysis (Figure 2) combining the more realistic bank geometry analysed by Osman and Thorne (1988) with the descriptions of pore water and hydrostatic confining pressures suggested by Simon *et al.* (1991). Darby and Thorne (1996a) used bank failure data from a total of 51 eroding sites in northern Mississippi to show that the new stability analysis provided improved predictions of planar-type bank failures compared to some previous analyses (Table I). Comparisons of predicted and observed data were obtained using geotechnical properties representing both 'ambient' conditions at the time of measurement and 'worst-case' conditions encountered at the time of failure. Worst-case conditions are associated with the development of saturated soils after prolonged periods of rainfall or flooding. Figure 3 shows that although the Darby–Thorne analysis performs better than the others listed in Table I, the factor of safety predicted for each of the 51 individual banks in the database is in general greater than the critical value of 1·0 or less associated with failing banks. This overprediction of factor of safety (underprediction of instability) may be related to two limitations with the model and/or data used in the predictive assessment. First, the database includes some stream banks with relatively low values of bank angle, which is known to cause problems for planar-type failure analyses (see below). Second, the database does not include any information about pore pressure conditions at the time of failure, so the contributing effect of positive pore pressure in generating bank instability is not accounted for in the assessment.

A computer software package based on the Darby and Thorne (1996a) stability analysis is presented in this paper. The computer program accounts for the geotechnical characteristics of the bank materials, the shape of the bank profile, and the positions of the groundwater and surface water elevations in order to estimate stability with respect to mass failure along a planar failure surface. Bank stability can be modelled either in terms of a factor of safety, or probability of failure. The computer program can also be used to determine the amounts of bed degradation and bank-toe erosion required to bring an initially stable slope to the point of failure. A key feature of the software is a Graphical User Interface (GUI) designed to facilitate data input and analysis of the results. The operation of the software is described in this paper, and some examples to illustrate potential applications are also provided.

LIMITATIONS AND SCOPE

Bank failures are often characterized by non-planar failure surfaces, including rotational (e.g. Bishop, 1955) and cantilever (e.g. Thorne and Tovey, 1981) type failures, as well as sapping type failures which involve the movement of groundwater (e.g. Hagerty, 1991) (Figure 4). In a discussion of the work by Darby and Thorne (1996a), Millar and Quick (1997) questioned the validity of the planar type stability analysis, noting that the predictive ability of planar-type analyses rapidly declines as the bank angle decreases (Bishop, 1955; Millar and Quick, 1997). Circular failure analyses are more appropriate for banks that are not steep. It is stressed that the analysis presented here is for steep (>60°) riverbanks which fail along planar surfaces, often involving the development of a tension crack. Such failures are common along the banks of streams that have been subjected to severe bed degradation (Darby and Thorne, 1997). Although the focus of the work discussed herein is on streambanks, the stability analysis should also be valid for any planar-type mass failure characterized by the slope geometry shown in Figure 2. The computer program might, therefore, be useful in analysing the stability of other steep slopes. Examples include the fronts of eroding salt marshes in estuarine environments, as well as coastal cliffs or hillslopes in general. However, we have thus far only used the software for applications in river environments. Our analysis also does not account for the stabilizing influence of negative pore water pressures in the unsaturated part of the bank (Rinaldi and Casagli, 1999; Simon *et al.*, 1999).

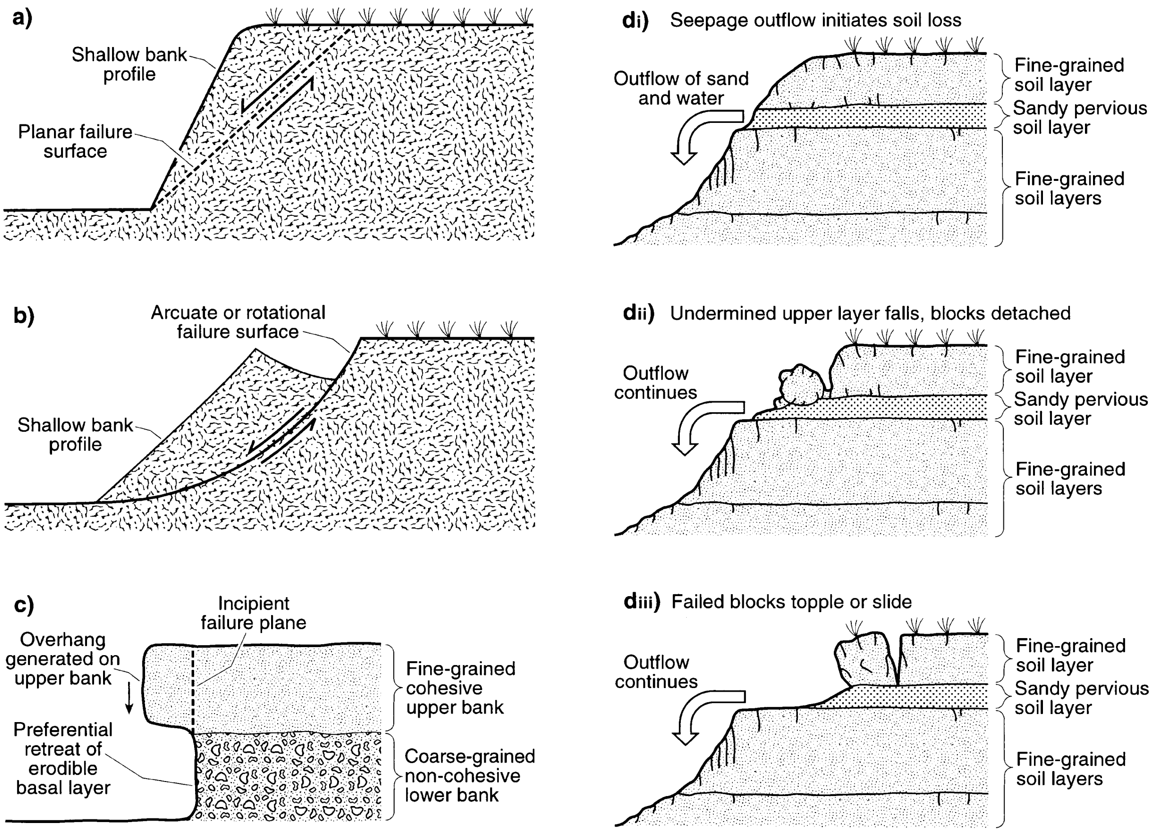


Figure 4. Examples of different types of bank failure mechanisms. (a) Planar failures as analysed herein; (b) rotational failures can be analysed using Bishop (1955); (c) cantilever failures can be analysed using Thorne and Tovey (1981); (d) piping/sapping type failures (modified after Hagerty, 1991)

Millar and Quick (1997) have also highlighted some errors in some of the equations published in Darby and Thorne (1996a). Specifically, some of the expressions for the hydrostatic uplift and hydrostatic confining forces (see figures 5 and 7 in Darby and Thorne (1996a)) were incorrectly written. These errors (Millar and Quick, 1997; Darby and Thorne, 1997) have been corrected in the software described in this paper. For completeness, correct versions of these expressions are listed here in Tables II and III.

DATA REQUIREMENTS

A range of data is required to use the bank stability analysis software presented herein. Channel geometry data are needed to characterize the shape of the bank profile (see Table IV and Figure 2). The software is designed to enable the user to provide the requisite bank profile data from one of two sources. If measurements of the bank profile are available these data may be entered directly into the program. Alternatively, the software is capable of extracting the requisite bank profile data automatically from HEC-2 hydrographic survey data files (US Army Engineer Hydrologic Engineering Center, 1982), though care must be taken to ensure that estimated bank profile values obtained using this option are realistic (see below). An advantage of this latter approach is that survey data for a long channel reach can be stored and manipulated in a single file, and incorporated into an integrated channel analysis. For example, the same data file can be used for (1) bank stability analysis using this software, (2) analysis of water surface profiles using the HEC-2 model (US Army Engineer Hydrologic Engineering Center, 1982), or (3) prediction of erosion and

Table II. Equations used to calculate the hydrostatic uplift force in the bank stability analysis. These expressions are corrected versions of the solutions presented in figure 5 of Darby and Thorne (1996a). The solution code identifies equations and figures in Darby and Thorne (1996a)

Solution code	Selection criteria	Equation for hydrostatic uplift force (U)
5A	$GWSE \leq y_f$	$U = 0$
5B	$y_f < GWSE \leq y_s$; $GWSE \leq y_k$	$U = \gamma_w \left[\frac{(GWSE - y_f)^2}{2 \sin \beta} \right]$
5CDE	$y_s < GWSE \leq y_k$; $y_f \leq y_s < y_k$; $GWSE \leq y_t$	$U = \gamma_w \left[\frac{(GWSE - y_f)^2}{2 \tan \beta} - \frac{(GWSE - y_s)^2}{2 \tan \alpha} \right] \frac{1}{\cos \beta}$
	$y_{fp} \geq GWSE > y_t$; $y_t > y_s \geq y_f$; $y_{fp} = y_k$	$U = \gamma_w \left[\left(\frac{(GWSE - y_f)^2}{2 \tan \beta} - \frac{(GWSE - y_s)^2}{2 \tan \alpha} \right) + \frac{(GWSE - y_t)^2}{2 \tan \alpha} \right] \frac{1}{\cos \beta}$
5IJK	$y_t = y_{fp}$; $y_s \geq y_f$; $GWSE > y_k \geq y_s$	$U = \gamma_w \left[\left(\frac{(GWSE - y_f)^2}{2 \tan \beta} - \frac{(GWSE - y_s)^2}{2 \tan \alpha} \right) - \frac{(GWSE - y_k)^2}{2 \tan \beta} \right] \frac{1}{\cos \beta}$
5FGH	$GWSE > y_k$; $GWSE > y_t$; $y_{fp} > y_k > y_s \geq y_f$; $y_{fp} > y_t > y_s \geq y_f$	$U = \gamma_w \left[\frac{(GWSE - y_f)(y_k - y_f)}{\tan \beta} - \frac{(y_k - y_f)^2}{2 \tan \beta} - \left(\frac{(GWSE - y_s)^2 - (GWSE - y_t)^2}{2 \tan \alpha} \right) \right] \frac{1}{\cos \beta}$
	$y_t = y_{fp}$; $y_s \geq y_f$; $GWSE > y_s$; $GWSE > y_k$	$U = \gamma_w \left[\left(\frac{(GWSE - y_f)^2 - (GWSE - y_k)^2}{2 \tan \beta} - \frac{(GWSE - y_s)^2}{2 \tan \alpha} \right) \right] \frac{1}{\cos \beta}$
	$y_{fp} \geq GWSE > y_t$; $y_t > y_s \geq y_f$; $y_k > y_s$	$U = \gamma_w \left[\left(\frac{(GWSE - y_f)^2}{2 \tan \beta} - \frac{(GWSE - y_s)^2}{2 \tan \alpha} \right) + \frac{(GWSE - y_t)^2}{2 \tan \alpha} \right] \frac{1}{\cos \beta}$
	$y_f < GWSE \leq y_s$; $GWSE > y_k$	$U = \gamma_w \left[\frac{(GWSE - y_f)(y_k - y_f)}{\tan \beta} - \frac{(y_k - y_f)^2}{2 \tan \beta} \right] \frac{1}{\cos \beta}$
	$y_f < GWSE \leq y_t$; $GWSE > y_k$; $y_s \geq y_f$	$U = \gamma_w \left[\frac{(GWSE - y_f)(y_k - y_f)}{\tan \beta} - \frac{(y_k - y_f)^2}{2 \tan \beta} - \left(\frac{(GWSE - y_s)^2}{2 \tan \alpha} \right) \right] \frac{1}{\cos \beta}$
	$GWSE > y_k$; $GWSE > y_t$; $y_t > y_s > y_k > y_f$	$U = \gamma_w \left[\left(\frac{(GWSE - y_f)^2 - (GWSE - y_k)^2}{2 \tan \beta} - \frac{(GWSE - y_s)^2}{2 \tan \alpha} \right) + \frac{(GWSE - y_t)^2}{2 \tan \alpha} \right] \frac{1}{\cos \beta}$

sedimentation using HEC-6 (US Army Engineers Hydrologic Engineering Center, 1977). In addition to bank profile data, the user must also supply the geotechnical characteristics of the bank materials. Clearly, the accuracy of the input data used in the bank stability analysis is vital in obtaining reliable predictions. It is, therefore, necessary to consider appropriate methods of data acquisition.

Information required to characterize the shape of the slope profile is best obtained from direct measurement during site surveys, but hydrographic survey data are frequently the only data available, particularly if analysis of historical conditions is required. Thorne and Abt (1993) caution that bank geometry data obtained from hydrographic surveys are often unreliable. This is because field crews often avoid the steepest bank profiles because they cannot be walked on and are hazardous to survey. The data entry interface is designed to facilitate extraction of bank profile data from HEC-2 input files. This is discussed further in the Appendix.

For the geotechnical properties of the bank materials, useful archive sources include results of test borings for foundations, especially during bridge construction. However, there is no substitute for data obtained

Table III. Equations used to calculate the hydrostatic confining force in the bank stability analysis. These expressions are corrected versions of the solutions presented in figure 7 of Darby and Thorne (1996a). The solution code identifies equations and figures in Darby and Thorne (1996a)

Solution code	Selection criteria	Equation for hydrostatic confining force (F_{cp})	Equation for angle ω	Equation for angle i
	$WSE \le y_f$	$F_{cp} = 0$	N/A	N/A
7F	$y_t = y_{fp}$; $y_s = y_f$	$F_{cp} = \frac{\gamma_w}{2\sin\alpha}\left(WSE - y_f\right)^2$	N/A	$i = \alpha - \beta$
7CDL	$y_f < WSE \le y_s$	$F_{cp} = \frac{\gamma_w}{2}\left(WSE - y_f\right)^2$	$\omega = 0$	$i = 90 - \beta$
7AGHJ	$y_{fp} \ge WSE \ge y_t > y_s > y_f$	$F_{cp} = \sqrt{\left[\frac{\gamma_w}{2}\left(WSE - y_f\right)^2\right]^2 + \left[\gamma_w\left(\frac{(y_t - y_s)^2}{2\tan\alpha} + \left\{(WSE - y_t)\left(\frac{y_t - y_s}{\tan\alpha}\right)\right\}\right)\right]^2}$	$\omega = \tan^{-1}\left[\frac{\gamma_w\left(\frac{(y_t-y_s)^2}{2\tan\alpha}+\left\{(WSE-y_t)\left(\frac{y_t-y_s}{\tan\alpha}\right)\right\}\right)}{\frac{\gamma_w}{2}(WSE-y_f)^2}\right]$	$i = 90 - (\beta + \omega)$
7BEI	$y_{fp} \ge y_t > WSE > y_s > y_f$	$F_{cp} = \sqrt{\left[\frac{\gamma_w}{2}\left(WSE - y_f\right)^2\right]^2 + \left[\frac{\gamma_w (WSE - y_s)^2}{2\tan\alpha}\right]^2}$	$\omega = \tan^{-1}\left[\frac{(WSE-y_s)^2}{\tan\alpha(WSE-y_f)^2}\right]$	$i = 90 - (\beta + \omega)$
7GK	$y_{fp} > WSE > y_t$; $y_s = y_f$	$F_{cp} = \sqrt{\left[\frac{\gamma_w}{2}\left(WSE - y_f\right)^2\right]^2 + \left[\gamma_w\left(\frac{(y_t - y_s)^2}{2\tan\alpha} + \left\{(WSE - y_t)\left(\frac{y_t - y_s}{\tan\alpha}\right)\right\}\right)\right]^2}$	$\omega = \tan^{-1}\left(\frac{\left[\gamma_w\left(\frac{(y_t-y_s)^2}{2\tan\alpha}+\left\{(WSE-y_t)\left(\frac{y_t-y_s}{\tan\alpha}\right)\right\}\right)\right]}{\left[\frac{\gamma_w}{2}(WSE-y_f)^2\right]}\right)$	$i = 90 - (\beta + \omega)$
	$y_{fp} > y_t$; $y_s = y_f$	$F_{cp} = \frac{\gamma_w}{2\sin\alpha}\left(WSE - y_f\right)^2$	N/A	$i = \alpha - \beta$

Table IV. Summary of input data used in bank stability analysis. Symbols are defined on Figure 2

Variable and Parameter Constraints	Symbol	Units	Comments
Floodplain elevation	y_{fp}	m	Obtained from hydrographic survey data
Elevation of base of bank	y_f	m	Elevation of the bank toe
Elevation of base of uneroded slope $y_{fp} \leq y_s \leq y_f$	y_s	m	Sometimes referred to as the base of the 'Upper Bank' (Simon and Hupp, 1992)
Elevation of base of relic tension crack $y_{fp} \leq y_t \leq y_f$	y_t	m	Sometimes referred to as the base of the 'Vertical Face' (Simon and Hupp, 1992). May or may not be present
Elevation of base of tension crack $y_{fp} \leq y_k \leq y_f$	y_k	m	May or may not be present
Angle of uneroded slope $\alpha \geq 50°$	α	°	
Bank material cohesion	c	N m^{-2}	
Bank material friction angle	ϕ	°	
Bank material unit weight	γ	N m^{-3}	
Groundwater surface elevation*	*GWSE*	m	Used to estimate hydrostatic uplift force
Water surface elevation*	*WSE*	m	Used to estimate hydrostatic confining force
Toe erosion calibration parameter*	*FEP*		Calibration parameter used in sensitivity tests (must be greater than zero)

* Optional data

during on-site surveys. Soil unit weight values can be obtained by standard laboratory analysis of samples taken from the field. Cohesion and friction angle values are best measured using *in situ* testing devices such as the Iowa Borehole Shear Tester (BST). Handy and Fox (1967), Luttenegger and Hallberg (1981), Thorne *et al.* (1981) and Simon and Hupp (1992) have reviewed the operation and features of the BST, and these details are not repeated here. Compared to conventional laboratory analyses, which require removal of a sample from the field, the main advantage of the BST is that the data obtained are representative of undisturbed bank material conditions. The BST provides estimates of the drained strength at the time of testing and, unless an independent measurement of pore water pressure is available, it is not possible to specify the cohesion and friction angle in terms of effective stress. Care has to be taken to ensure that the conditions under which the geotechnical parameter values were actually measured are known, as this affects the type of stability analysis (total stress versus effective stress analysis) that can be conducted.

Geotechnical properties of bank materials vary through time in response to changes in soil moisture content. It is, therefore, usually necessary to conduct sensitivity analyses to assess changing stability conditions under 'ambient' and 'worst-case' conditions. The term 'ambient' refers to values of cohesion, friction angle and unit weight measured under drained conditions in the field at the time of site visits. The term 'worst-case' refers to estimated values corresponding to conditions encountered when the soil is fully saturated, as might occur after prolonged or heavy rainfall or after periods of inundation. Under these conditions, the soil unit weight is maximized due to high water content, while cohesion and friction values are reduced to account indirectly for the effects of positive pore water pressure. In the limit, the friction angle may be artificially reduced to zero (Thorne and Abt, 1993). This is a total stress stability analysis and, if this approach is adopted, care must be taken not to use the pore water pressure option available with this software, to avoid double accounting of pore pressure effects.

Elevations of the water surface in the channel and the groundwater table in the bank can optionally be specified by the user to compute the hydrostatic confining and uplift forces, respectively, for use in the effective stress stability analysis (Figure 2). Pore water pressure effects can be significant controls on bank stability, but if these options are chosen care must be taken to ensure that values of cohesion and friction angle appropriate for an effective stress analysis are used.

The computer analysis is also able to determine the relative amounts of bed degradation and/or bank-toe erosion required to bring an initially stable bank to the point of failure. This is done through use of the fluvial erosion parameter (*FEP*) in sensitivity tests. The *FEP* expresses the ratio of fluvial shear erosion to bed degradation at the toe of the bank. When this option of the computer program is selected (see below for further details), the bank profile is progressively deformed by adding small increments of bed scour (ΔZ) and bank-toe erosion (ΔW) in proportions set by the value of *FEP*, until limiting conditions of stability are reached. By varying the magnitude of *FEP*, the user can investigate how different combinations of lateral and vertical erosion, governed by various geomorphic controls, lead to varying bank stability responses. For example, in selecting values of the calibration parameter, very cohesive banks that are resistant to fluvial shear erosion will have small values of *FEP*. Conversely, larger values of *FEP* are appropriate for weak, erodible banks.

EXAMPLE APPLICATION: BANK STABILITY ON THE UPPER MISSOURI RIVER IN MONTANA

Example applications are now provided to illustrate some of the various ways in which the software can be used to analyse bank stability. Three different approaches to assessing bank stability are described herein. These include analysis of worst-case versus ambient soil conditions (total stress stability analysis); use of the pore water and hydrostatic confining pressure analyses to simulate the effects of rapid drawdown on bank stability (effective stress stability analysis); and use of the probabilistic bank stability analysis. In each case the examples are based on data obtained from a field site located on the Upper Missouri River in Montana.

In September 1995 a field reconnaissance study of 250 km of the Upper Missouri River between Fort Peck Dam and its confluence with the Yellowstone River indicated that approximately 50 per cent of the riverbanks in the study reach exhibited evidence of recent geotechnical failure and instability (Pokrefke *et al.*, 1998). Riparian landowners and users had expressed concern that construction of Fort Peck Dam and its subsequent operation for hydropower generation may have resulted in accelerated bank erosion. As part of a study to address these concerns (Pokrefke *et al.*, 1998), the Darby and Thorne (1996a) bank stability analysis was used to assess the extent to which bank stability conditions have changed since construction of the dam in 1936. Full details of the simulations are provided in Pokrefke *et al.* (1998), but for the purposes of this paper data are presented for just one site. The study site is located on the north bank some 225 km downstream of Fort Peck Dam, some 10 km upstream of the US Geological Survey (USGS) stream gauge at Culbertson, Montana. At this site net changes in bank stability conditions during the period 1955 to 1995 were assessed by extracting bank profile data from US Army Corps of Engineers channel surveys (see Table V). Geotechnical data (Table V) used in this example were provided by personnel from the US Department of Agriculture. Agricultural Research Service, National Sedimentation Laboratory, Oxford, Mississippi, who obtained borehole test data at the site in September 1996.

Ambient versus worst-case soil conditions

Since geotechnical parameter values vary as a function of soil moisture content, it is usually necessary to conduct stability analyses using different geotechnical parameter values representing two cases, for ambient and worst-case conditions (e.g. Thorne *et al.*, 1981; Simon and Hupp, 1992; Darby and Thorne, 1996a). The term 'ambient' refers to the geotechnical characteristics measured in the field under drained conditions at the time of site visits. The term 'worst-case' corresponds to undrained conditions when the soil is fully saturated, as might occur after prolonged or heavy rainfall, or after prolonged inundation. To illustrate the importance of analysing bank stability for each of these conditions, two analyses were performed using the Upper Missouri River data (Table V). In this type of total stress analysis, where the effects of pore water pressure conditions for failure under worst-case conditions are accounted for indirectly by modifying the values of c, γ and ϕ, the

Table V. Input data used in example applications. Elevations are expressed in metres above an arbitrary local datum. Geotechnical data for 'ambient' conditions were obtained from 'drained' borehole tests. Geotechnical data for 'worst-case' conditions are estimates

Input data	Upper Missouri River (1955)	Upper Missouri River (1995)
Bank profile data		
Floodplain elevation (m)	5·0	5·0
Bank-toe elevation (m)	1·04	1·34
Bank angle (degrees)	57	83
Elevation of base of uneroded slope (m)	1·04	1·34
Elevation of base of vertical face (m)	5·0	5·0
Elevation of base of tension crack (m)	5·0	5·0
Fluvial erosion parameter	0·05	0·05
Geotechnical data: ambient conditions		
Cohesion (kPa)	14·9	14·9
Friction angle (degrees)	29	29
Unit weight (kN m^{-3})	20·9	20·9
Geotechnical data: worst case conditions		
Cohesion (kPa)	14·9	14·9
Friction angle (degrees)	0	0
Unit weight (kN m^{-3})	21·5	21·5
Water surface elevations used for pore water and hydrostatic confining pressure analysis		
Drawdown conditions: groundwater surface elevation (m)	--	3·5
Drawdown conditions: water surface elevation (m)	--	1·4
Non-drawdown conditions: groundwater surface elevation (m)	--	1·4
Non-drawdown conditions: water surface elevation (m)	--	3·5

pore water pressure analysis option must not be used, to avoid double-accounting of pore water pressure effects.

Initially using data for the 1955 bank profile, the software provides an estimate that the factor of safety of the bank was 2·42 under ambient conditions. Since the factor of safety (FS) is defined as the ratio of resisting to driving forces acting on the incipient failure block, values of FS > 1 represent stable bank conditions, while values of FS < 1 indicate unstable bank conditions, with FS $= 1$ indicating critical bank conditions. Hence the 1955 bank profile is predicted to be stable under ambient conditions. With the fluvial erosion parameter set to a value of 0·05, a further 4·10 m of bed degradation, together with $0{\cdot}05 \times 4{\cdot}10 = 0{\cdot}205$ m of bank-toe erosion, would be required to destabilize this bank. In contrast, the factor of safety computed using data for the 1995 bank profile was 1·40 under ambient conditions.

Repeating the analysis to compare the results obtained for ambient conditions with those for worst-case conditions, it is found that reducing the soil shear strength and increasing the soil unit weight results in the banks becoming much less stable with respect to mass failure. For example, predicted factor of safety values decline from 2·42 to 1·29 (1955), and from 1·40 to 0·86 (1995), under ambient and worst-case conditions, respectively. There is therefore a clear decrease in bank stability over time, with stable banks in 1955 and with banks prone to failure under worst-case conditions in 1995. These modelling results are consistent with observations by riparian landowners and users that bank erosion has accelerated in recent years. The predicted decrease in bank stability appears to be related to a net change in bank morphology in the period between 1955 and 1995 (Table V). Although there has been a stabilizing tendency related to deposition that has increased the elevation of the bank toe, this has been offset by an increase in bank slope. This suggests that bank instability has been triggered by direct (lateral) fluvial erosion rather than undermining through bed level lowering. The change in bank stability conditions is significant enough to be of concern to riparian

landowners and users. For example, the dimensions of the failure block associated with the unstable (1995) bank are considerable, with up to 3·7 m of floodplain loss resulting in 6·8 m^3 of sediment delivered to the channel per unit length of unstable bank.

Use of the pore-water and hydrostatic confining pressure analyses

The Upper Missouri study reach experiences a highly regulated flow regime, with discharge releases from Fort Peck Dam varying in response to hydropower releases. There is a concern that high rates of bank erosion may be exacerbated by these variable releases. This is because banks might become saturated during high discharge releases. Subsequent rapid drawdown could lead to a perched water table, generation of excess pore pressures and removal of hydrostatic confining forces (Simon *et al.*, 1991).

Using data for the 1995 bank profile, the possible adverse effects of rapid drawdown on bank stability were simulated in an effective stress stability analysis by selecting the pore water pressure and confining pressure options. The elevations of maximum and minimum water surface elevations associated with hydropower release cycles were estimated using water surface profile data for peak and base flows obtained from US Corps of Engineers surveys (see Table V). These water surface elevations were then used to estimate the relative positions of the perched groundwater surface (which was assumed equivalent to the maximum water surface elevation) and the drawdown water surface (which was assumed equivalent to the minimum water surface elevation), respectively (Pokrefke *et al.*, 1998). In this effective stress analysis the 'ambient', rather than 'worst-case', values of soil cohesion and friction angle (Table V) were used to avoid double-accounting for the effects of pore water pressure.

The results indicate that the factors of safety predicted for drawdown and non-drawdown conditions are 1·21 and 1·89, respectively. These values compare to a factor of safety of 1·40 predicted using the total stress stability analysis described above. These results illustrate that positive pore water pressures generated during rapid drawdown do indeed reduce the overall stability of the bank, but in this example the decrease in stability is not sufficient to trigger mass failure. In fact, repeat simulations using the bank stability software indicate that for this example a groundwater surface elevation of around 4·45 m would be required to generate positive pore water pressures sufficiently large to trigger mass failure. This suggests that much larger fluctuations in flow elevation than occur under the present flow regime are required to destabilize the bank through increased positive pore pressures. Hence, while riparian landowners concerned with bank erosion along the Upper Missouri may be correct in observing increased bank instability along the reach over time, they are probably incorrect in attributing this to fluctuations in water level associated with hydropower generation. Instead, lateral erosion and consequent steepening of the bank profile during the period 1955 to 1995 appears to be responsible for the observed instability.

Probabilistic bank stability analysis

An alternative to using the factor of safety as an index of bank stability is to present the modelling results in terms of the probability of failure. Huang (1983) developed a probabilistic bank stability analysis to provide a rational basis for analysing the statistical risk of slope failure, whereas Darby and Thorne (1996b) used a probabilistic bank stability analysis as a basis for estimating the streamwise extent of bank instability along discrete modelling reaches. In both cases the probability of failure was determined by representing the soil cohesion, friction angle and unit weight parameters used in the governing bank stability equation (see Figure 2) with probability distributions. Hence, estimates of probability of failure are obtained using measured probabilities of occurrence of bank material properties (see Simon (1989) for an example of a probability distribution).

The bank stability analysis software described in this paper has the capability to perform probabilistic-based bank stability analyses. However, while a probabilistic approach is useful in many practical applications, it must be recognized that the level of data acquisition required to generate the requisite probability distributions to represent the material properties is quite high. Indeed, in demonstrating the application of the probabilistic bank stability analysis, the example data files provided with this software contain arbitrarily derived probability functions. These functions were derived by assuming approximately

Table VI. Simulated probability of failure for a range of groundwater surface elevation values. Based on data representing the Upper Missouri River 1995 bank profile (see Table V) and geotechnical data provided in the example data files

Groundwater surface elevation (m)	Simulated probability of failure
1·0	0·426
2·0	0·426
3·0	0·438
4·0	0·438
4·45	0·537
4·95	0·554

normal distributions about the 'modal' values of cohesion, friction angle and unit weight obtained along the Missouri River under ambient conditions.

Using the example data files provided, together with the 1955 and 1995 Upper Missouri bank profile data listed in Table V, the probabilistic analysis can be selected by checking the requisite box on the main data entry screen (see Appendix). Using the default soils data, which represents ambient conditions, the computed probability of failure values are found to be 0·255 and 0·426 for the 1955 and 1995 profiles (Table V), respectively. These low probability of failure values are consistent with the relatively high factor of safety estimates (2·42 and 1·40, respectively) obtained previously.

To develop a feel for how the probability of failure varies in response to changing soil moisture conditions, a series of simulations can be undertaken using the 1995 bank profile data together with the soils data provided in the example data files. In these simulations, which are analogous to those undertaken in the preceding effective stress analysis, the groundwater surface elevation is progressively increased to determine the effects of increasing positive pore water pressure on simulated probability of failure. The results (Table VI) show that the probability of failure progressively increases as the bank material becomes progressively more saturated. At near total saturation (groundwater surface elevation of 4·95 m), the simulated probability of failure is 0·554. Direct comparison with the preceding effective stress analysis is not possible because of the arbitrarily derived probability distributions used in this example. Nonetheless, as a very rough aid to interpretation, it is useful to highlight the preceding analysis in which a groundwater surface elevation of around 4·45 m was required to generate bank instability (factor of safety less than unity). Table VI indicates that this particular groundwater elevation value leads to a predicted probability of failure that exceeds 0·5, so this value may provide a guide that the simulated bank is prone to failure. The precise relationship between factor of safety and probability of failure is a topic of current research.

CONCLUSIONS

The software described in this paper takes account of the geotechnical characteristics of the bank materials, the shape of the bank profile, and the relative positions of the ground and surface water elevations to estimate stability with respect to mass failure along a planar-type failure surface. Results can be displayed either in terms of a factor of safety (ratio of resisting to driving forces), or probability of failure. The computer analysis is able to determine the relative amounts of bed degradation and bank-toe erosion that are required to bring an initially stable bank to the point of failure. Data for the analysis are supplied directly in the form of user-supplied bank profile and geotechnical data. In addition, the program can automatically extract initial estimates of the requisite bank profile data from HEC-2 hydrographic survey data files. Examples are provided to illustrate the use of the program using data from the Upper Missouri River in Montana. The examples demonstrate the effects of worst-case versus ambient geotechnical parameter values on bank stability, the sensitivity of bank stability to changes in the magnitudes of fluvially driven bank-toe erosion, and the destabilizing effects of rapid drawdown on bank stability.

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APPENDIX: OBTAINING AND USING THE SOFTWARE

The software described in this paper can be used with 32-bit Windows operating systems (e.g. Windows 95, Windows 98, Windows NT). Hardware requirements include a maths coprocessor, a mouse, and sufficient

hard disk storage space for installation and storage of results files. The modelling software can be obtained by downloading a self-extracting installation program from the following Internet address:

http://fritham.geog.soton.ac.uk/users/darbyse/bankstab/download.htm

Instructions on how to use the installation program to unpack and register the modelling software and example data files are provided on this Internet site.

Once the modelling software has successfully been installed, the program can be executed using the file named bankstb9_1.exe. Some users may need to alter their screen and/or desktop resolution to size program windows to their screen. Clicking on the 'Continue' button displayed on the welcome screen allows the user to proceed to the main data entry screen. At this point the user must choose whether to characterize bank profiles using either automatic data extraction from a HEC-2 style input file, or by entering bank profile data directly into the boxes shown on the data entry screen. HEC-2 files can be selected by choosing an appropriate data file and clicking 'OK', otherwise opt for direct data entry by clicking the 'Cancel' button. In the example files loaded by the installation software, the file named rb.dat is a HEC-2 data file containing 1992 hydrographic survey data along a reach of Red Banks Creek in northern Mississippi.

Bank profile data are entered via the main data entry screen. The following variables are used to define the bank profile (see Table VI and Figure 2).

- The *floodplain elevation* (y_{fp}) is the highest point on the bank profile.
- The *elevation of the bank toe* is the lowest point on the bank profile.
- The *bank angle* (α) is measured along the uneroded portion of the bank slope. This slope is the segment of the bank profile between the 'vertical face' and the region of the bank toe subject to lateral fluvial shear erosion.
- The *base of the uneroded slope* (y_s) marks the break point between the uneroded bank profile and a near-vertical face that corresponds to an erosional front generated by fluvial shear erosion of the toe. In the absence of fluvial shear erosion, the base of the uneroded surface is equal to the elevation of the bank toe.
- The *base of the vertical face* (Simon and Hupp, 1992; y_t on Figure 2) may or may not be present. The 'vertical face' corresponds to a remnant tension crack generated during previous episodes of bank failure.
- The *elevation of the base of the tension crack* (y_k) marks the location of any known tension cracks that are present behind the bank face. Users should note that, independent of the value of y_k entered, the computer program automatically constrains the tension crack depth so that it is always less than half the total bank height. This prevents computational problems associated with unrealistically deep tension cracks.

Bank geometry data entry proceeds either by filling in the requisite parameter values in the data entry boxes (for user-supplied bank profiles) or by clicking on the 'Next Section' button (for use with HEC-2 data files) in the Options box on the right-hand side of the data entry screen. For those applications where the analysis is concerned only with a single bank profile (rather than for both the left- and right-hand banks), data can be entered in the left bank column only.

Using HEC-2 data files

If a HEC-2 input data file (e.g. rb.dat) is selected, clicking on the 'Next Section' button displays left and right bank geometry data extracted from individual cross-sections of the HEC-2 hydrographic survey data file. Clicking on the 'Next Section' button allows the user to scroll through the file in forward sequence. Once the chosen cross-section is selected, the 'Computations' and 'Draw Section' buttons on the Options menu are activated. At this stage, the user should check to see if the bank geometry values extracted by the program from the HEC-2 data file are realistic. To assist the user in this respect, click on the 'Draw Section' button to graph the selected cross-section. On these graphs, black lines indicate the HEC-2 hydrographic survey data while the purple lines indicate the extracted bank profile, for both left and right banks. By modifying the values of data entered in the highlighted left and right bank data boxes on the data entry screen, the user can ensure that the extracted and measured bank profiles match. Typically, the user will need to adjust the value of the bank angle to account for the tendency of bank angle to be underestimated by field survey.

'Soil Properties' button

After editing the bank profile data, the user can click on the 'Soil Properties' button located on the Options menu to edit the geotechnical parameters via a new data entry screen used for bank material characteristics. This data screen displays the values for cohesion, unit weight and friction angle in the form of average (modal) values for the non-probabilistic bank stability analysis (located in the bottom left of the screen). Probabilistic distributions used for the probabilistic bank stability analysis, in the form of three data tables, are located along the top half of the screen.

When modifying the probabilistic frequency distributions for the cohesion, unit weight and friction angle (if the probabilistic analysis is selected), care should be taken to ensure that the sum of probability values is 100 per cent. Care should also be taken to update the modal values of each distribution in the average soil property value boxes in the bottom left of the screen. The soil probability distributions can be displayed graphically by clicking on the relevant button(s) in the 'Show Graphs' box located at the bottom right of the screen. Any changes are saved by clicking the 'Store New Data' button. Return to the main level data entry screen by clicking on the 'End' button. The 'Reset' button allows the user to reset values to those last stored during editing.

Selecting program options

Program options are selected to suit the requirements of the analysis. A maximum of four options are available which allow users to analyse various aspects of riverbank stability.

1. Choose the value of the fluvial erosion parameter ($\Delta W/\Delta Z$). This parameter is used in conjunction with the non-probabilistic (modal) bank stability analysis only. The fluvial erosion parameter must have a value greater than zero and separate values can be used for left and right banks.
2. Conduct analysis of pore pressure effects by clicking on the 'Pore Water Pressure' box. If the analysis is selected, the elevations of the groundwater table are entered in the activated data boxes, independently for left and right banks. The pore water pressure analysis should only be used in an effective stress analysis, with appropriate values of soil cohesion and friction angle.
3. Conduct analysis of confining pressure effects by clicking on the 'Confining Pressure' box. If the analysis is selected, the elevation of the water in the channel is entered in the activated data entry box.
4. Select the analysis method by choosing either the non-probabilistic or probabilistic bank stability analyses by clicking on the appropriate data entry box.

Performing computations

Once all the input data and program options have been entered and carefully checked, the program is executed by clicking on the 'Computations' button located in the Options menu on the main level data entry screen. Warning messages are displayed if the chosen bank angle values are sufficiently low that a planar failure is unlikely. A warning message is also displayed if the right bank profile data column has been left blank, in which case the program uses the values entered in the left blank profile data column. Computations are normally completed within a few seconds.

Viewing the results

After program execution is successfully completed, results files for left and right banks (darbyl.out and darbyr.out) are written to the home directory of the software package (C:\newbank). These files are overwritten in each successive simulation, so users should rename these files in order to archive the results. At the end of an individual simulation, the results are also displayed on a results window within the graphical user interface. To display the results, it is necessary to click on the 'Continue' button in the Options menu, having allowed sufficient time for the simulations to be completed.

Error messages

During some simulations using the non-probabilistic stability analysis, an error message 'Error – too much lateral erosion' may be displayed. This error occurs when stable banks require an excessively large amount of bed degradation and lateral toe erosion to generate mass failure. Specifically, the lateral erosion required is sufficient to generate an overhanging bank profile. The model described herein cannot simulate this type of bank profile and computations are, therefore, terminated. Reducing the value of the fluvial erosion parameter used in the computations can sometimes circumvent the error.

